International Journal of Civil Engineering (IJCE)
ISSN(P): 2278-9987; ISSN(E): 2278-9995
Vol. 3, Issue 2, Mar 2014, 103-110


International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

# DYNAMIC RESPONSES OF SQUARE TLP'S TO RANDOM WAVE FORCES

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## ABSTRACT

Tension leg platform (TLP) is a suitable type for very deep-water oil production. The TLP is a compliant structure behaving like a floating one. This paper investigates the nonlinear response of the Square TLP configuration due to a random sea waves. Random waves were generated according to Pierson-Moskowitz spectrum and acts on the structure in the surge direction. The hydrodynamic forces evaluation is based on the modified Morison equation. Coupling effect and added mass are considered in the developing of the equation of motion. The nonlinear equation of motion is solved in the time domain utilizing the modified Euler scheme. Time history responses, phase planes, and Power spectrum densities (PSD) for the nonlinear responses are shown. Since only uni-directional waves in the surge direction was considered in the analysis, surge, heave and pitch degrees-of-freedom responses are influenced significantly.

**KEYWORDS:** Compliant Structures, Random Sea Wave Forces, Nonlinear Response

## INTRODUCTION

TLP's are floating flexible (compliant) structures with six degrees of freedom; (surge, sway, heave, roll, pitch and yaw) see figure 1. They undergo relatively long period of vibration associated with the motion in the horizontal plan. The inherited large deformations in their horizontal plan, causes nonlinearity in the structure stiffens matrix.

Abou-Rayan et al. [2014, 2013a, 2013b, and 2012] have investigated the dynamic responses for a square and triangular TLP configurations under uni-directional regular waves in the surge direction of the platform. They also have considered the coupling effect between all six degrees of freedom. They found that, the coupling has no effect on the surge or the heave responses and has insignificant effect on the pitch response. Also, from the phase plane responses they concluded that, that the steady state behavior of the TLP is periodic and stable. Under a combination of regular waves and an impulse load subjected to a triangular TLP configuration and varying water depths, Chandrasekaran et al. [2007] have investigated its dynamic responses. To determine the dynamic responses of square and triangular TLPs subjected to random waves, Kurian et al. (2008) have developed a numerical study. They found that the responses of triangular TLPs are much higher than those of square TLP, in agreement with Abou-Rayan [2014]. Low [2009] presented a formulation for the linearization of the tendon restoring forces of a TLP. He found that, the linearization technique facilitates accurate predictions of the mean offsets and the response variances, including the slow-drift component. Tabeshpour et al., have investigated the dynamic responses of a square TLP configuration under unidirectional random waves.

In this investigation, a numerical scheme has been developed to investigate the nonlinear dynamic response characteristics of a square TLP configuration under random waves. The time history of random wave is generated based on Pierson-Moskowitz spectrum and acts on the structure in the surge direction. The hydrodynamic forces are calculated using the modified Morison equation according to Airy's linear wave theory. Utilizing modified Euler equation step-by-step integration technique, the solution for the equation of motion was obtained.

## EQUATION OF MOTION

The equation of motion is coupled and nonlinear and can be written as, Abou-Rayan et al. [2012]:

$$[M]\{x^{\bullet\bullet}(t+\Delta t)\}+[C]\{x^{\bullet}(t+\Delta t)\}+[K]\{x(t+\Delta t)\}=\{F\ (t+\Delta t)\} \tag{1}$$

Where [***M***] is the structure mass matrix; **[*C*]** is the structure damping matrix; **[*K*]** is the structure stiffness matrix; and {*F (t+Δt)*} is the hydrodynamic force vector. $x, \dot{x}, \textbf{and}\ \ddot{x}$ are the structural displacement vector, the structural velocity vector, and the structural acceleration vector respectively. Time history of the Random wave is generated based on Pierson-Moskowitz spectrum. Utilizing Morison's equation and using Airy's linear wave theory, wave forces are calculated at the instantaneous equilibrium position of the TLP acting in the surge direction. Added mass coefficients in the mass matrix and coupling effect were considered in the model. Wave force coefficients, $C_d$ (Drag coefficient= 1) and $C_I$ (Inertia coefficient=2), are the same for the pontoons and the columns and are independent of frequencies as well as constant over the water depth. To solve the equation of motion a step-by-step integration scheme was developed based on the modified Euler method. The analysis was carried out for dynamic response in both time and a frequency domain. The mathematical model derived in this investigation is based on the TLP model studied by Abou-Rayan et al. [2012], for detailed description of the hydrodynamic data and the geometric properties and the derivation of the equation of motion, the reader is referred to aforementioned reference.

## REPRESENTATION OF WAVE SPECTRA

Researchers have studying ocean waves have proposed several formulations for wave spectra dependent on a number of parameters (such as wind speed, fetch, or modal frequency). One of the simplest descriptions for the energy distribution is the Pierson-Moskowitz (P-M) spectrum. It is an empirical relationship that defines the distribution of energy with frequency within the ocean. It assumes that if the wind blows steadily for a long time over a large area, then the waves will eventually reach a point of equilibrium with the wind. This is known as a fully developed sea. Pierson and Moskowitz developed their spectrum from measurements in the North Atlantic during 1964, and presented the following relationship between energy distribution and wind.

$$S_{\eta\eta}(\omega) = \frac{H^2 T_e}{8\pi^2}\left(\frac{T_e\omega}{2\pi}\right)^{-5} \exp\left[-\frac{1}{\pi}\left(\frac{T_e\omega}{2\pi}\right)^{-4}\right] \tag{2}$$

Where $H$ is the wave height in m, $T_e$ is energy period in sec and ω is the angular frequency. The normalized Pierson-Moskowitz spectrum for $H$ =15 m and $T_e$ =15sec is shown in Figure 2. Real random waves are not sinusoidal. However, they can be represented with a good approximation as superposition of regular waves. The sea surface elevation is given by:

$$\eta(x,t) = \lim \sum_{n=1}^{N} A_n \cos\left(k_n x - \omega_n t + \alpha_n\right) \tag{3}$$

$$A_n = \sqrt{2S_\eta(\omega_n)\omega_0}\ ,\ \omega_n = n\omega_0$$

Where $A_n$ is the amplitude of the nth component wave, $\omega_n$ is the wave frequency of the *n*th component wave, $k_n$ is the wave number of the *n*th component wave, $x$ is the horizontal distance from the origin , $\alpha_n$ is the random phase angle of the *n*th component wave, and $S_\eta(\omega)$ *i*s the one-sided power spectrum for sea surface elevation. Time histories of the water particle velocity and acceleration are computed by wave superposition according to Airy's linear wave theory,

utilizing the sea surface elevation time history $\eta(x,t)$. Accordingly, the horizontal water particle velocity $\dot{u}(x,t)$, the vertical water particle velocity $\dot{v}(x,t)$, the horizontal water particle acceleration $\ddot{u}(x,t)$ and the vertical water particle acceleration $\ddot{v}(x,t)$ are given by, respectively:

$$\dot{u}(x,t) = \sum_{n=1}^{N} A_n\, \omega_n \cos\,(k_n x - \omega_n t + \alpha_n)\; \frac{\cosh\,(k_n y)}{\sinh\,[k_n(d+\eta)]} \tag{4}$$

$$\dot{v}(x,t) = \sum_{n=1}^{N} A_n\, \omega_n \sin(k_n x - \omega_n t + \alpha_n) \frac{\sinh(k_n y)}{\sinh[k_n(d+\eta)]} \tag{5}$$

$$\ddot{u}(x,t) = \sum_{n=1}^{N} A_n\, \omega_n^2 \sin\,(k_n x - \omega_n t + \alpha_n)\; \frac{\cosh\,(k_n y)}{\sinh\,[k_n(d+\eta)]} \tag{6}$$

$$\ddot{v}(x,t) = \sum_{n=1}^{N} A_n\, \omega_n^2 \sin\,(k_n x - \omega_n t + \alpha_n)\; \frac{\sinh\,(k_n y)}{\sinh\,[k_n(d+\eta)]} \tag{7}$$

Where, y is vertical distance at which the wave kinematics is considered, and d is the water depth. A typical random sea surface elevation is shown in Figure 3.

## RESULTS AND DISCUSSIONS

A numerical scheme was developed using MATLAB software where solution based modified Euler method was obtained. Wave forces were taken to be acting in the direction of surge degree-of-freedom. A square (66mX66m), Abou-Rayan [2012], TLP in 600 m deep water was considered for the numerical study. The geometric properties are: diameter of pontoon, $D_p$ = 9m, diameter of columns $D_c$ = 18m, and the tether total force =160000 KN. Table 1 shows the coupled natural time periods of the structure. It is observed that TLPs have very long period of vibration associated with motions in the horizontal plane (say 60 to 100 seconds). Since typical wave spectral peaks are between 6 to 15 seconds, resonant response in these degrees of freedom is unlikely to occur.

### Surge Response

The time history of the surge response (*H =15m, $T_e$ =15sec.*) for the TLPs configuration is shown in Figure 4. The surge response has considerably bigger values (8m to -8m) than those for the same configuration under regular waves (5m to -5m); see Abou-Rayan [2012]. It is clear that the response is not a pure periodic one; on the other hand it is not a chaotic one. Despite the fact that the excitation is random, but the response is semi-periodic. The Phase plane, Figure 7, shows that the response is oscillating between tow semi-periodic motions one is bigger than the other, relatively. To get an insight into this behavior, the response spectra for wave height of 15 m and wave period of 15sec. was obtained and the results are shown in Figure 10. It is clear that we have a peak centered around a frequency of 0.2 rad/sec. That frequency is far away from the natural frequency of the square configuration, which is 0.064 rad/sec which exclude resonance phenomenon.

### Heave Response

The time history of the Heave response (*H =15m, $T_e$ =15sec.*) for the TLPs configuration is shown in Figure 5. The heave response has lower values than those for the surge one, since the excitation is in the surge direction. The Phase plane, Figure 8, shows that the structure is responding in the same pattern as in the surge one. Again, the response is oscillating between tow semi-periodic motions one is bigger than the other, relatively. Also, the response has semi-periodic pattern. To get an insight into this behavior, the response spectra for wave height of 15 m and wave period of 15sec. was

obtained and the results are shown in Figure 11. It is clear that we have a peak centered around a frequency of 0.2 rad/sec. Therefore, a resonance response is unlikely to occur, since the natural frequency for that degree of freedom is 2.83 rad/sec.

**Pitch Response**

The time history of the pitch response ($H=15m,\ T_e=15sec.$) for the TLPs configuration is shown in Figure 6. From pitch response, shown in figure 9, it is clear that the response follows the same behavior pattern as in the surge and heave responses but with lesser response values than those of the surge response. Which, is expected since the random waves are uni-directional in the surge direction. Figure 12 shows the PSD for the response indicating that the maximum peak response is centered around 0.21 rad/sec. Finally, other responses, sway, roll, and yaw, are very small in the order of $x10^{-6}$. This indicates that these degrees of freedom have not been affected by the uni-directional wave in surge. Therefore, their responses (figures) are not shown.

## CONCLUSIONS

The present study investigates the dynamic response of a square TLP under random wave forces in the surge direction considering all degrees of freedom of the system. A numerical scheme was developed where Morison's equation according to Airy's linear wave theory was used. Analyses were carried out in both time and frequency domains. The time history of random wave is generated based on Pierson-Moskowitz spectrum and it acts on the structure in the surge direction. Results for the time histories, phase planes, and power density spectrum for the affected degrees of freedom have been presented. Based on the results shown in this paper, the following conclusions can be drawn:

- TLP's have very long period of vibration (60 to 100 seconds) associated with motions in the horizontal plane. Since typical wave spectral peaks are between 6 to 15 seconds, resonant response in these degrees of freedom is unlikely to occur.
- The phase plane and the response spectra show that the steady state behavior of the structure is semi-periodic and stable.
- For the orientation of the TLP and for the unidirectional wave considered, translational (surge and heave) and rotational (pitch) degree-of-freedom (DOF) responses are influenced significantly. Variations in water particle kinematics with water depth, induced forces and moments activate in all six DOF, but the response is predominant in only three DOF.

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## APPENDICES

**Table 1: Calculated Natural Structural Periods**

| DOF | | | | | |
|---|---|---|---|---|---|
| Surge | Sway | Heave | Roll | Pitch | Yaw |
| 97.099 | 97.099 | 2.218 | 3.126 | 3.126 | 86.047 |

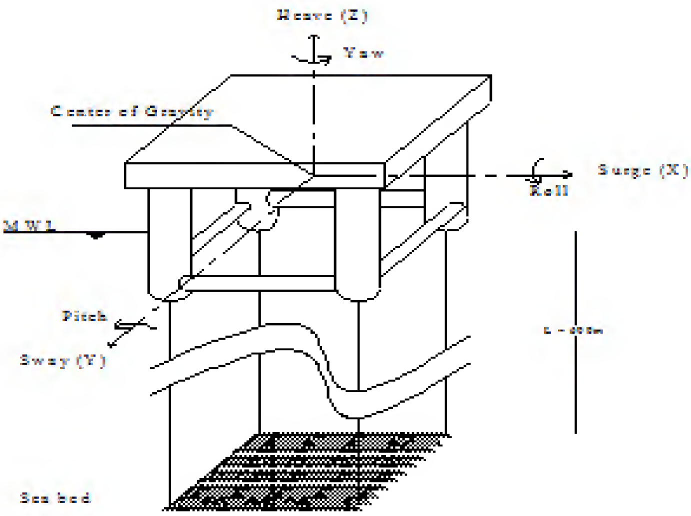


**Figure 1: Degrees-of-Freedom of the Platform**

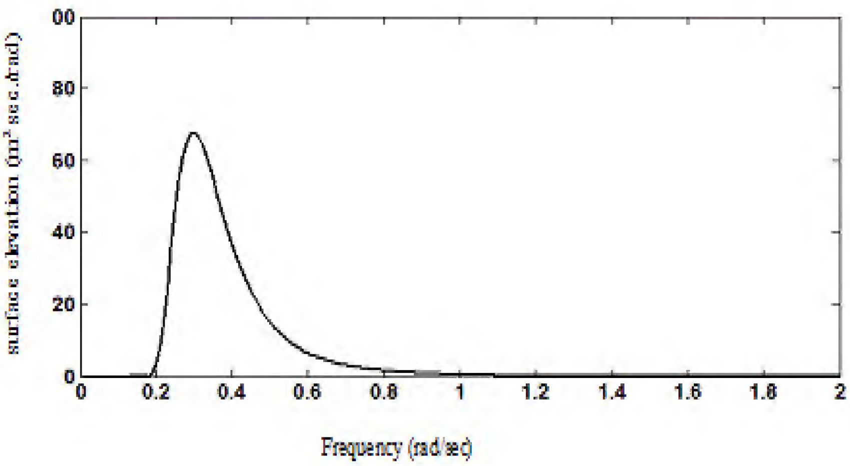


**Figure 2: Peirson-Moskowitz PSD ($H_s$ =15m, $T_s$ =15Sec.)**

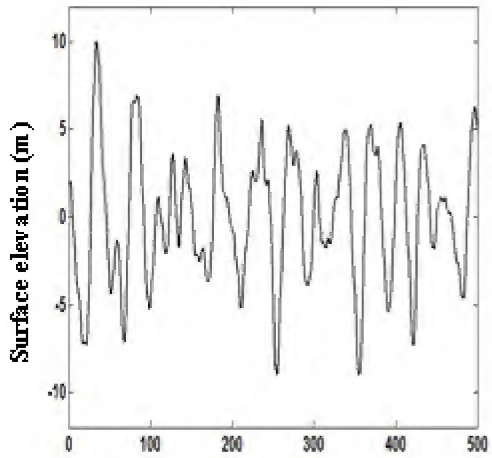


Figure 3: Random Elevation of Surface Wave

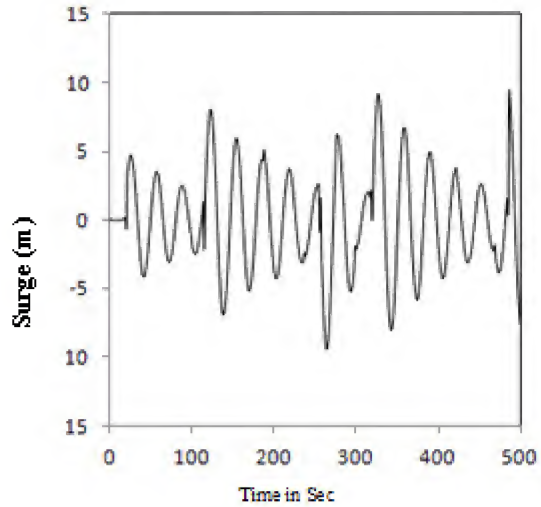


Figure 4: Surge Displacement Response

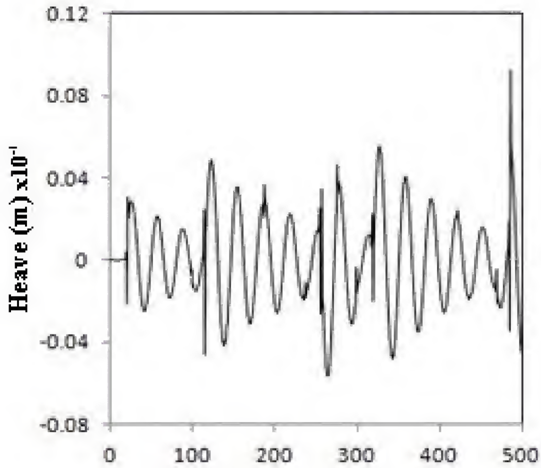


Figure 5: Heave Displacement Response

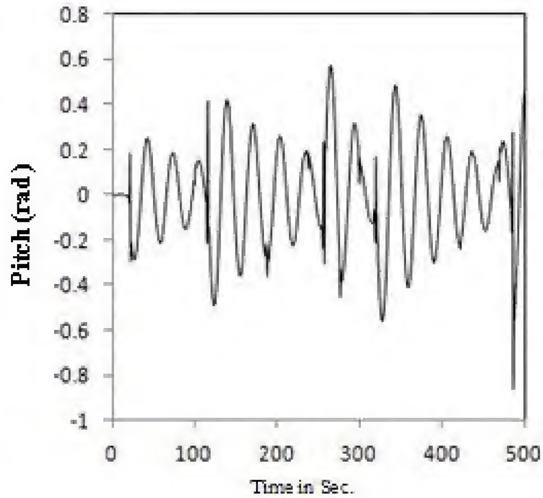


Figure 6: Pitch Displacement Response

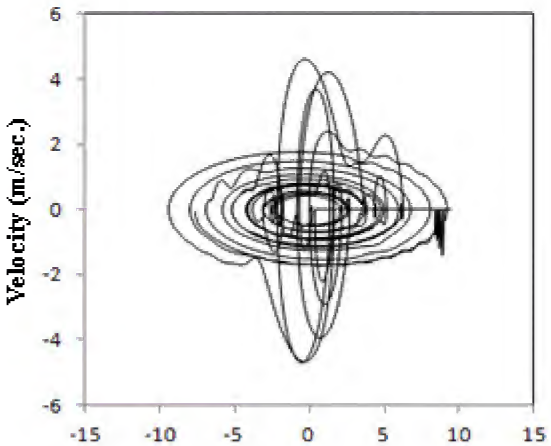


Figure 7: Phase Plane for the Surge Response

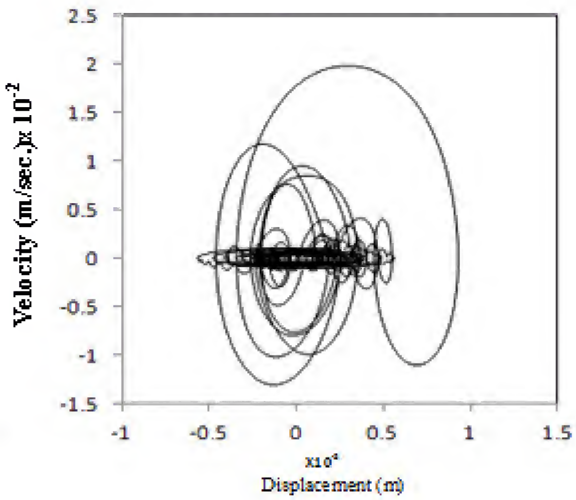


Figure 8: Phase Plane for the Heave Response

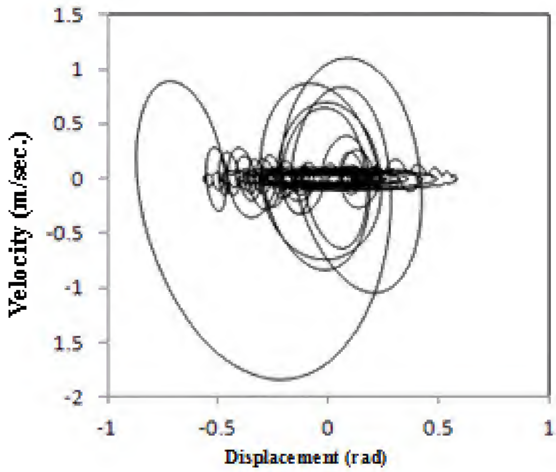


Figure 9: Phase Plane for the Pitch Response

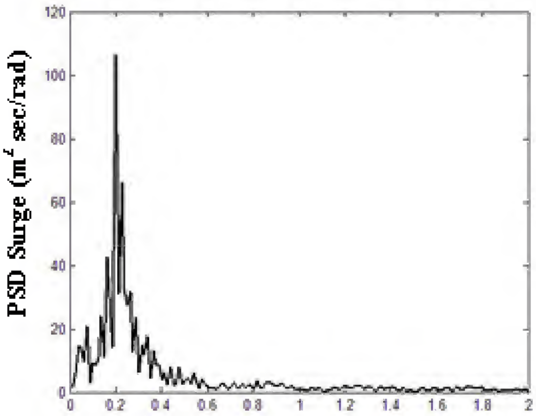


Figure 10: PSD of Surge Displacement

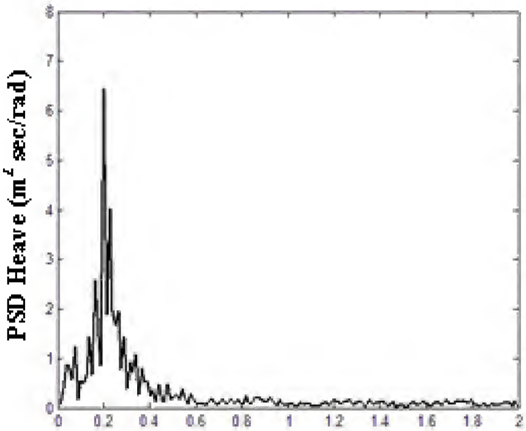


Figure 11: PSD of Heave Displacement

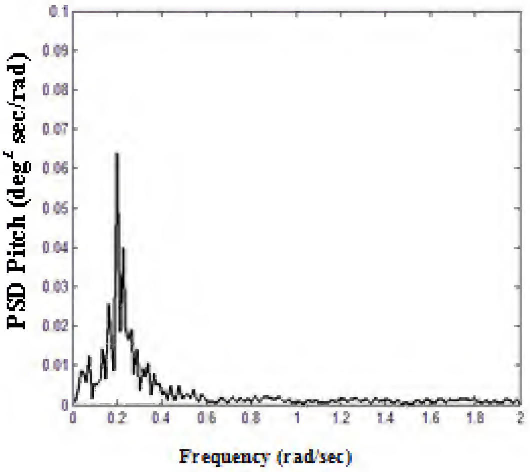


Figure 12: PSD of Pitch Displacement